PART 1

1.3 Segment: Functions; Graphs of Power Functions

1.3.1 Functions

Suppose $s$ is the setting on an industrial oven dial and $T$ is the internal oven temperature. We would have pairs $(s, T)$ that dial settings and temperatures that occur at those settings. The relationship between $s$ and $T$ would be modelled by an equation in $s$ and $T$.

If the model was a general equation, such as $900s^2 - T^2 = -1600$, then setting the dial at 1 would give temperatures according to $900 - T^2 = -1600$, $T^2 = 2500$, $T = \pm 50$. Setting the dial at 1 could produce a temperature of $-50$ or a temperature of 50. In this model, the dial setting does not determine exactly one temperature; $T$ is not a function of $s$.

We desire a model where the dial setting determines exactly one oven temperature; $T$ is a function of $s$. Such a model might be $T = 30s^2 + 50$ where $T$ is a function of $s$; each value of $s$ determines a value for $T$.

Given an equation for $x$ and $y$, $y$ is a function of $x$ if each value of $x$ determines exactly one value for $y$. If $y$ is a function of $x$ then any vertical line will intersect the graph for the equation of $x$ and $y$ in at most one point. Since, $y$ is a single value determined by $x$, we write $y = f(x)$ where $f(x)$ is equal to an expression in $x$. 

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Some functions are given in function form, e.g., \( y = 3x^2 - 3 \). Other functions are given as an equation, but not in function form, e.g., \( 3x - 5y = 13 \). We may write \( 3x - 5y = 13 \) with \( y \) as a function of \( x \): \( y = \frac{3x - 13}{5} \).

**Example 10**

\[
f(x) = 3x^2 - 2x + 1
\]
\[
f(2) = 3(2)^2 - 2(2) + 1 = 9
\]
\[
f(x + 2) = 3(x + 2)^2 - 2(x + 2) + 1 = 3x^2 + 10x + 9
\]
\[
f(\Delta x) = 3(\Delta x)^2 - 2(\Delta x) + 1
\]
\[
f(x + \Delta x) = 3 (x + \Delta x)^2 - 2 (x + \Delta x) + 1 = 3x^2 + 6x\Delta x + (\Delta x)^2 - 2x - 2\Delta x + 1
\]
\[
f(x + \Delta x) - f(x) = (3x^2 + 6x\Delta x + (\Delta x)^2 - 2x - 2\Delta x + 1) - (3x^2 - 2x + 1)
\]
\[
= 6x\Delta x + (\Delta x)^2 - 2\Delta x
\]
\[
\frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{6x\Delta x + (\Delta x)^2 - 2\Delta x}{\Delta x} = \frac{\Delta x(6x + \Delta x - 2)}{\Delta x} = 6x + \Delta x - 2.
\]

The *domain* of \( y = f(x) \) is the set of values \( x \) for which \( f(x) \) is defined. For \( y = 3x^2 - 6x + 5 \), \( 3x^2 - 6x + 5 \) is defined for all real numbers \( x \), so the domain is the set of all real numbers. In interval notation, the set of real numbers can be written as \((-\infty, \infty)\). For \( y = \sqrt{x - 1} \), \( \sqrt{x - 1} \) is defined when \( x - 1 \geq 0, x \geq 1 \), so the domain is the set of real number greater than or equal to one. In interval notation, the set of number where \( x \geq 1 \) can be written \([1, \infty)\). When \( x \) is in the domain of \( y = f(x) \), there
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is a point on the graph either above or below the value $x$ on the $x$-axis.

The range of $y = f(x)$ is the set of values $y$ that can be computed from $f(x)$.

For $y = 3x^2 - 6x + 5$, the graph is a parabola that opens upward from the vertex at $x = 1$. So $y = f(1) = 2$ is in the range and any $y \geq 2$ is in the range (in interval notation $[2, \infty)$). For $y = \sqrt{x - 1}$, $y = \sqrt{x - 1} \geq 0$ and $0 = \sqrt{1 - 1}$. The range is the set of numbers greater than or equal to zero (in interval notation $[0, \infty)$).

We obtain the inverse of $y = f(x)$ by interchanging $x$ and $y$: $x = f(y)$ and solve for $y$ to get $y = g(x)$. A test for inverses is the fact that $f(g(x)) = x = g(f(x))$.

Example 11

For $x \geq 1$, $y = f(x) = x^2 + 1$ and $y = g(x) = \sqrt{x - 1}$ are inverses.

Check: $f(g(x)) = f(\sqrt{x - 1}) = (\sqrt{x - 1})^2 + 1 = x - 1 + 1 = x$ and
$g(f(x)) = g(x^2 + 1) = \sqrt{x^2 + 1 - 1} = x$.

Example 12

Find the inverse of $y = 3x - 7$.

Interchange $x$ and $y$: $x = 3y - 7$.

Solve for $y$: $x = 3y - 7$, $3y - 7 = x$, $3y = x + 7$, $y = \frac{x + 7}{3}$. 
1.3.2 Power Functions and Graphs

A power function is a function of the form $y = cx^n$ where $c$ is a non-zero constant and $n$ is real number. Some examples with $c = 1$ are $y = x^3$, $y = x^4$, $y = x^{1/3}$, $y = x^{-2}$.

When $n = 0$, $y = c$ is a horizontal line.

When $n = 1$, we have already seen that $y = cx$ is a straight line with slope $c$ and passing through $(0, 0)$.

We have also seen that, when $n = 2$, $y = cx^2$ is a parabola with vertex $(0, 0)$ opening upward if $c > 0$ and downward if $c < 0$.

When $n$ is a whole number greater than two, for $x \geq 0$, $y = cx^n$ has a shape similar to a parabola. For $x \leq 0$, $y = cx^n$ is a reflection of the graph when $x \geq 0$. When $n$ is even the reflection is oriented the same—up or down, when $n$ is odd the reflection is opposite—up or down.
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When \( n \) is a negative whole number, the graph of \( y = x^n \) is similar to a hyperbola is shape. The graph has no intercepts, but approaches the \( x \)-axis when \( x \) becomes larger (\( y = 0 \) is a horizontal asymptote) and approaches the \( y \)-axis when \( y \) becomes larger (\( x = 0 \) is a vertical asymptote).

\[
y = x^{-3} = \frac{1}{x^3}
\]

Now consider \( y = x^{1/m} \) where \( m \) is a positive whole number greater than one. Let's use \( m = 3 \) for the moment. Then \( y = x^{1/3} = \sqrt[3]{x} \) (the cube root of \( x \)). Also, the inverse is \( x = y^{1/3} \), \( x^3 = (y^{1/3})^3 = y \). The graph of \( y = x^{1/3} \) is a reflection of the graph.
of $y = x^3$ through a $45^\circ$ line.

Example 13

Graph $y = -x^5$

The graph is a reflection of $y = x^5$ and has the basic shape of $y = x^3$. Two points should be sufficient to guide our graph:
Example 14

Graph $y = (x - 2)^5 - 3$

The graph is a duplicate of $y = x^5$ with the point $(0, 0)$ moved to
Example 15

Graph $y = \sqrt{x + 2}$.

The graph is a duplicate of $y = \sqrt{x}$ with the point $(0,0)$.
moved to $(-2, 0)$.

The graph of $y = |f(x)|$ is obtained from the graph of $y = f(x)$ by reflecting any part of the graph below the $x$-axis through the $x$-axis.
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\[ y = 5 + 7x - 2x^2 \]

\[ y = |5 + 7x - 2x^2| \]