2.4 Segment: Exponential and Logarithmic Functions (pre-requisite)

We have looked at exponents, powers, and power functions. Power functions are of the form \( y = x^n \) where the base \( x \) is the variable and the exponent \( n \) is a fixed value.

Now we are again going to define a class of functions using exponents and powers, but
PART 2

the base $b$ is a fixed value and the exponent $x$ is the variable, an exponential function $y = b^x$.

**Example 7**

$y = 3^x$

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>100</th>
<th>$-1$</th>
<th>$-2$</th>
<th>$-100$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>1</td>
<td>3</td>
<td>9</td>
<td>$5.154 \times 10^{47}$</td>
<td>$1/3$</td>
<td>$1/9$</td>
<td>$1.940 \times 10^{-48}$</td>
</tr>
</tbody>
</table>

Characteristics of $y = 3^x$ to be noted:

1. the values of $y$ are positive

2. the values of $y$ become extremely close to zero as the values of $x$ become very large negative numbers—the line $y = 0$ is an asymptote

3. the values of $y$ become extremely large as the values of $x$ become very large
There several numbers that occur naturally in many real settings, but those numbers are not whole numbers or fractions. One example in the number π that represents the ratio of the diameter to the circumference of a circle. Another such number is the natural number e that is related to the ratio of the amount of pressure in a container to the rate at which the contents leak out of the container. The number e is approximately 2.7183.

Example 8

The graph of \( y = e^x \) is similar to the graph of \( y = 3^x \).
Example 9

The graph of $y = 2e^x + 1$ has the shape of $y = e^x$ with a horizontal
asymptote $y = 1$.

The inverse of the graph of $y = 3^x$ is given by the graph of $x = 3^y$.

The graph of $x = 3^y$ is also a function, this inverse function is a logarithm $y = \log_3 x$ with base 3. The expression $\log_3 x$ can be read "log base 3 of $x$". There are...
two cases that have special notation, $\log_{10} x$ is written $\log x$ and $\log_e 10$ is written $\ln x$ and is called the natural log.

**Example 10**

The logarithmic form of $5 = w^4$ is $4 = \log_w 5$.

The exponential form of $5 = \log_3 t$ is $3^5 = t$.

The fact that $y = \log_b x$ is the inverse of $y = b^x$ gives rise to several rules for logarithms:

- $b^{\log_b x} = x$
- $\log_b b^x = x$
- $\log_b v + \log_b w = \log_b (vw)$
- $\log_b v - \log_b w = \log_b \left(\frac{v}{w}\right)$
- $n \log_b x = \log_b x^n$
- $\log_e x = \frac{\log_b x}{\log_b e}$

**Example 11**

$$e^{\ln(x-1)} = x - 1$$

$$\log 10^t = t$$
PART 2

\[ \log 2t + \log 3t = \log 6t^2 \]

\[ \log_3 9t - \log_3 3t = \log_3 3 \]

\[ 3 \ln 2y = \ln 9y^2 \]

Calculators and computers only have functions for computing log and ln. To compute a value, such as $\log_3 5.2$, use either $\frac{\log 5.2}{\log 3}$ or $\frac{\ln 5.2}{\ln 3}$.

**Example 12**

Solve $3^{2x-1} = 9$ for $x$.

\[ \log_3 3^{2x-1} = \log_3 9, \ 2x - 1 = 2, \ 2x = 3, \ x = 1.5. \]

**Example 13**

Solve $4^{x-1} = 3^{3-2x}$ for $x$.

\[ \ln 4^{x-1} = \ln 3^{3-2x}, \ (x-1) \ln 4 = (3-2x) \ln 3, \ x \ln 4 - \ln 4 = 3 \ln 3 - 2x \ln 3, \ x \ln 4 + 2x \ln 3 = 3 \ln 3 + \ln 4, \ (\ln 4 + 2 \ln 3)x = 3 \ln 3 + \ln 4, \ x = \frac{\ln 4 + 2 \ln 3}{3 \ln 3 + \ln 4} \approx 0.765. \]

**Example 14**

Solve $5 = 3 \log(1 - 2x)$ for $x$.

\[ 10^5 = 10^{3 \log(1 - 2x)}, \ 10^5 = 10^{\log(1 - 2x)^3}, \ 10^5 = (1 - 2x)^3, \]

\[ \sqrt[3]{10^5} = 1 - 2x, \ x = \frac{1 - \sqrt[3]{10^5}}{-2} \approx -22.71. \]

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