\[ \lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h \to 0} \frac{45 - 45}{h} = \lim_{h \to 0} 0 = \lim_{h \to 0} 0 = 0. \] The derivative of a constant function is zero.

Example 11

Find the derivative of \( y = 5x - 6. \)

\[ \lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h \to 0} \frac{5(x+h)-6 - (5x-6)}{h} = \lim_{h \to 0} \frac{5x+5h-6-5x+6}{h} = \lim_{h \to 0} \frac{5h}{h} = \lim_{h \to 0} 5 = 5. \] The derivative of a linear function is the slope of the line.

4.3 Segment: Power Rule for Derivatives

4.3.1 Finding the Derivative of a Power Function.

Let us investigate the derivative of a function of the form \( f(x) = x^n, \) a power function.

Try \( f(x) = x^2. \) \( \frac{df}{dx}(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^2-x^2}{\Delta x} = \lim_{\Delta x \to 0} \frac{x^2+2x(\Delta x)+(\Delta x)^2-x^2}{\Delta x} = \lim_{\Delta x \to 0} \frac{2x(\Delta x)+(\Delta x)^2}{\Delta x} = \lim_{\Delta x \to 0} 2x + (\Delta x) = 2x. \)

Try \( f(x) = x^3. \) \( \frac{df}{dx}(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^3-x^3}{\Delta x} = \lim_{\Delta x \to 0} \frac{x^3+3x^2(\Delta x)+3x(\Delta x)^2+(\Delta x)^3-x^3}{\Delta x} = \lim_{\Delta x \to 0} \frac{3x^2+3x(\Delta x)+(\Delta x)^2}{\Delta x} = \lim_{\Delta x \to 0} 3x^2 + 3x(\Delta x) + (\Delta x)^2 = 3x^2. \)

Try \( f(x) = x^4. \) \( \frac{df}{dx}(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^4-x^4}{\Delta x} = \lim_{\Delta x \to 0} \frac{x^4+4x^3(\Delta x)+6x^2(\Delta x)^2+4x(\Delta x)^3+(\Delta x)^4-x^4}{\Delta x} = \lim_{\Delta x \to 0} \frac{4x^3+6x^2(\Delta x)+4x(\Delta x)^2+(\Delta x)^3}{\Delta x} = 4x^3. \)
PART 4

Notice there is a pattern in the derivative expressions: the exponents and coefficients increase by one as the power function exponent increases. Furthermore, the derivative exponent is one less than the exponent of the power function and the derivative coefficient is the exponent of the power function.

Power Rule for Derivatives: If \( f(x) = x^n \), then \( \frac{df}{dx}(x) = nx^{n-1} \).

Example 12

\[ f(x) = x^7, \quad \frac{df}{dx}(x) = 7x^6. \]

Example 13

\[ y = x^{-3}, \quad \frac{dy}{dx}(x) = -3x^{-4}. \]

Example 14

\[ g(t) = \sqrt{t}, \quad g(t) = t^{\frac{1}{2}}, \quad \frac{dg}{dt}(t) = \frac{1}{2}t^{-\frac{1}{2}} = \frac{1}{2\sqrt{t}}. \]

4.3.2 Linearity of the Derivative

The derivative of a sum is the sum of the derivatives: \( \frac{d(f+g)}{dx}(x) = \frac{df}{dx}(x) + \frac{dg}{dx}(x) \).
PART 4

The derivative of a constant times a function is the constant times the derivative of a function: \( \frac{d(cf)}{dx}(x) = c \frac{df}{dx}(x) \), where \( c \) is a constant.

Example 15

\[ f(x) = 3x^2 - 2x + 4, \quad \frac{df}{dx}(x) = 6x - 2 + 0 = 6x - 2. \]

Example 16

\[ g(t) = \frac{4}{t^2} + 5t^3, \quad g(t) = 4t^{-2} + 5t^3, \quad \frac{dg}{dt}(t) = -8t^{-3} + 15t^2 = -\frac{8}{t^3} + 15t^2 \]

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