Part 5

Properties of Curves; Extrema, Concavity, Asymptotes, Intercepts

5.1 Segment: Increasing and Decreasing Functions

Informally, a function is increasing if the graph rises upward to the right and is decreasing if graph falls downward to the right.
Formally, a function $f$ is increasing on an interval $[a, b]$ if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$. A function $f$ is decreasing on an interval $[a, b]$ if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$.

Example 1
Let $f$ be given by the graph

Assume that the graph continues indefinitely to the left and right in the manner that it approaches the edge of the visible graph.

The function $f$ is increasing on the intervals $(-\infty, -2], [1, \infty)$. The function $f$ is decreasing on the interval $[-2, 1]$.

Example 2
Let $f$ be given by the graph

The function $f$ is increasing on the interval $(-1, \infty)$. The function $f$ is decreasing on the interval $(-\infty, -1)$.

5.1.1 Slopes of Curves and Increasing, Decreasing Functions

Notice that the slopes are positive for an increasing function and the slopes are negative for a decreasing function.
We can determine where a function is increasing or decreasing by determining where the derivative (slope) is positive or negative.

Test for increase, decrease using the derivative: \( y = f(x) \).

\[ f \text{ is increasing through } x \text{ if } f'(x) > 0, \]

\[ f \text{ is decreasing through } x \text{ if } f'(x) < 0. \]

To move forward with the inequalities \( f'(x) > 0 \) and \( f'(x) < 0 \), we need to take a sidetrack and review solving inequalities.

### 5.1.2 Solving Inequalities

Lets investigate solving inequalities using the function \( g(x) = x^2 + x - 6 \). Solve the inequality \( g(x) > 0 \).
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The first step to solving an inequality is to solve the related equality: \( g(x) = 0, x^2 + x - 6 = 0, (x + 3)(x - 2) = 0, x = -3, 2. \)

The second step is to note any values for where the graph of the function is discontinuous (possible piecewise continuous)—most likely these are values where the function is not defined. For \( g(x) = x^2 + x - 6 \) there are no discontinuities.

The possible places where \( g(x) = x^2 + x - 6 \) can change sign, change from \( g(x) > 0 \) to \( g(x) < 0 \), is at the values of \( x \) found in the first two steps. In this case \( x = -3, 2. \)

The third step is to divide the set of real numbers into intervals using \( x = -3, 2: \) \((-\infty, -3), (-3, 2), (2, \infty)\). Now pick one value of \( x \) in each of the intervals: \( x = -4, x = 0, x = 3. \)

The fourth step is to test the value of \( g(x) = x^2 + x - 6 \) at the chosen values:

\[
g(-4) = (-4)^2 + (-4) - 6 = 10 > 0, \ g \text{ is positive on the interval } (-\infty, -3),
\]

\[
g(0) = (0)^2 + (0) - 6 = -6 < 0, \ g \text{ is negative on the interval } (-3, 2),
\]

\[
g(3) = (3)^2 + (3) - 6 = 6 > 0, \ g \text{ is positive on the interval } (2, \infty).
\]

Thus \( g(x) = x^2 + x - 6 > 0 \) on the intervals \((-\infty, -3)\) and \((2, \infty)\).

Example 3

Solve \( \frac{2x-5}{x+2} < 0. \)

First: solve \( \frac{2x-5}{x+2} = 0, \ 2x - 5 = 0, \ x = \frac{5}{2}. \) Also, \( \frac{2x-5}{x+2} \) is undefined.
when \( x + 2 = 0 \), \( x = -2 \).

Second: the values \( x = -2, \frac{5}{2} \), divide the real numbers into the intervals \((-\infty, -2), (-2, \frac{5}{2}), (\frac{5}{2}, \infty)\).

Third: choose some value in each interval, say \( x = -3, 0, 3 \).

Fourth: test \( \frac{2x-5}{x+2} \),

\[
\frac{2(-3)-5}{(-3)+2} = \frac{-11}{1} = 11 > 0,
\]

\[
\frac{2(0)-5}{(0)+2} = \frac{-5}{2} < 0,
\]

\[
\frac{2(3)-5}{(3)+2} = \frac{1}{5} > 0.
\]

Thus, \( \frac{2x-5}{x+2} < 0 \) on the interval \((-2, \frac{5}{2})\).

\[5.1.3 \text{ Using } f'(x) \text{ to Determine where } f \text{ is Increasing and Decreasing} \]

Example 4

Determine the intervals on which the \( f(x) = 2x^3 + 3x^2 - 36x + 5 \) is increasing. Also, determine the intervals on which \( f \) is decreasing.

\( f'(x) = 6x^2 + 6x - 36 \). Solve \( 6x^2 + 6x - 36 = 0 \), \( 6(x+3)(x-2) = 0 \), \( x = -3, 2 \). These values divide the real numbers into the intervals \((-\infty, -3), (-3, 2), (2, \infty)\). Test \( f'(x) \).
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\[ f'(-4) = 36 > 0 \]
\[ f'(0) = -36 < 0 \]
\[ f'(3) = 54 > 0. \]

The function \( f(x) = 2x^3 + 3x^2 - 36x + 5 \) is increasing on the intervals \((-\infty, -3), (2, \infty)\) and decreasing on the interval \((-3, 2)\).