PART 5

\[ f'(-4) = 36 > 0 \]
\[ f'(0) = -36 < 0 \]
\[ f'(3) = 54 > 0. \]

The function \( f(x) = 2x^3 + 3x^2 - 36x + 5 \) is increasing on the intervals \((-\infty, -3), (2, \infty)\) and decreasing on the interval \((-3, 2)\).

5.2 Segment: Maximum or Minimum of a Function

The graph of a function may reach a highest point. The height of the graph at the highest point is the maximum of the function. Similarly, the height of the lowest point is the minimum of the function.

A point where there is a maximum, a curve has the nearby characteristics of the top of a hill. A point where there is a minimum, a curve has the nearby character-
We are going to look for points where a function has a relative maximum or minimum.

An extreme value of a function is either a maximum or a minimum. At a point where the graph of a function has an extreme value, the function has a horizontal tangent line; the slope is zero.

To find possible places where a function has an extreme value, we can find the place where the function has zero slope, i.e., where $f'(x) = 0$. The solutions to
PART 5

Figure 5.2:

\[ f'(x) = 0 \] are called the critical values for \( f \).

Points where \( f'(x) \) does not exist are also critical values for \( f \).

Let's find the critical values for \( f(x) = 2x^3 + 3x^2 - 36x - 81 \). Compute the derivative: \( f'(x) = 6x^2 + 6x - 36 \). Solve \( 0 = 6x^2 + 6x - 36 \), \( 0 = 6(x^2 + x - 6) \), \( 0 = 6(x + 3)(x - 2) \), \( x = -3, 2 \). If \( f \) has a maximum or a minimum it will occur at one of the critical values \( x = -3, 2 \).

If \( y = f(x) \) has maximum value at \( x_0 \) then \( y = f(x) \) increase to the left of \( x_0 \) and decreases to the right of \( x_0 \). The derivative will be positive on the right and negative on the left of a value where \( f \) has a maximum. Similarly, the derivative will be negative on the right and positive on the left of a value where \( f \) has a minimum.

For the function \( f(x) = 2x^3 + 3x^2 - 36x - 81 \), the critical values \( x = -3, 2 \)
divide the real numbers into intervals where $f'(x) > 0$ or $f'(x) < 0$.

Choose a value in each interval: $-4$, $0$, $3$. Test $f'(x)$ at each of these values:

$f'(-4) = 6(-4)^2 + 6(-4) - 36 = 36 > 0$, the derivative in positive,

$f'(0) = 6(0)^2 + 6(0) - 36 = -36 < 0$, the derivative in negative,

$f'(3) = 6(3)^2 + 6(3) - 36 = 36 > 0$, the derivative in positive.
The function $f(x) = 2x^3 + 3x^2 - 36x - 81$ has a relative maximum at $x = -3$ and a relative minimum at $x = 2$.

### 5.3 Segment: Concavity

Consider the straight lines $y = x$, $y = 2x$, and $y = \frac{1}{2}x$. All three lines have positive slopes and are increasing. But, the yield for an input value of $x$ varies for the three lines. For the line $y = x$, an input value of $x$ yields an output $y$ that is the same; you get out what you put in. For the line $y = 2x$, an input value of $x$ yields an output value $y$ that is twice the size; you double your value. For the line $y = \frac{1}{2}x$, an input value of $x$ yields an output value $y$ that is only half as large; you loose half your value.

If $y$ were profit and $x$ were a an investment to produce profit, all three lines represent increasing profit, but, $y = \frac{1}{2}x$ only returns half of what is invested, $y = x$ returns what is invested, and $y = 2x$ returns double what is invested. The slope of the line represents the rate of return on an investment; we wish for more than just increasing profit but a rate of return greater than one. Using straight line models the slope is constant. General curves $y = f(x)$ offer the possibility of increasing slope as well as an increasing function; an increasing rate of return.

A function $y = f(x)$ is **concave up** if the slope is increasing, i.e., the derivative $f'(x)$ is increasing. A function $y = f(x)$ is **concave down** if the slope is decreasing, i.e., the derivative $f'(x)$ is decreasing.