Part 6

Differentiation Rules

6.1 Segment: Product Rule for Derivatives

We could find the derivative of \( f(x) = (4x^2 + 3x - 5)(3 + 7x - 4x^2 - x^3) \) by first multiplying out the product and collecting like terms, but that could be a little tedious.

The Product Rule for Derivatives allow us to compute the derivative of \( f(x) \) by finding the derivatives of the factors \( 4x^2 + 3x - 5 \) and \( 3 + 7x - 4x^2 - x^3 \) seperately. The Product Rule for the Derivative of a product of two factors \( g(x) \) and \( h(x) \) is formally:

\[
\frac{d}{dx} (g(x)h(x)) = g(x) \frac{d}{dx} h(x) + h(x) \frac{d}{dx} g(x)
\]

For \( y = f(x) = (4x^2 + 3x - 5)(3 + 7x - 4x^2 - x^3) \) we have \( g(x) = 4x^2 + 3x - 5 \) and \( h(x) = 3 + 7x - 4x^2 - x^3 \). Also, \( g'(x) = \frac{d}{dx} g(x) = \frac{d}{dx} (4x^2 + 3x - 5) = 8x + 3 \) and \( h'(x) = \frac{d}{dx} h(x) = \frac{d}{dx} (3 + 7x - 4x^2 - x^3) = 7 - 8x - 3x^2 \). So \( f'(x) = \frac{d}{dx} (g(x)h(x)) = \)
\[ g(x) \frac{d}{dx} h(x) + h(x) \frac{d}{dx} g(x) = (4x^2 + 3x - 5)(7 - 8x - 3x^2) + (3 + 7x - 4x^2 - x^3)(8x + 3). \]

**Example 1**

\[ f(x) = (3 - 2x)(3x^2 - 4x) \]

\[ f'(x) = (3 - 2x) \left( \frac{d}{dx}(3x^2 - 4x) \right) + (3x^2 - 4x) \left( \frac{d}{dx}(3 - 2x) \right) = (3 - 2x)(6x - 4) + (3x^2 - 4x)(-2). \]

**Example 2**

\[ y = 2\sqrt{x}(4x^5 - 3x^3 - 9) \]

\[ y' = (2\sqrt{x}) \left( \frac{d}{dx}(4x^5 - 3x^3 - 9) \right) + (4x^5 - 3x^3 - 9) \left( \frac{d}{dx}(2\sqrt{x}) \right) = (2\sqrt{x})(20x^4 - 9x^2) + (4x^5 - 3x^3 - 9) \left( \frac{1}{\sqrt{x}} \right). \]

### 6.2 Segment: Quotient Rule for Derivatives

The derivative of a quotient of two factors, such as, \( f(x) = \frac{4x^2 + 2x - 3}{5 + 2x} \), can be computed from the derivatives of the factors using the Quotient Rule for Derivatives:

\[
\frac{d}{dx} \left( \frac{g(x)}{h(x)} \right) = \frac{h(x) \frac{d}{dx} g(x) - g(x) \frac{d}{dx} h(x)}{[h(x)]^2}
\]

For \( y = f(x) = \frac{4x^2 + 2x - 3}{5 + 2x} \), \( g(x) = 4x^2 + 2x - 3 \) and \( h(x) = 5 + 2x \). Also, \( g'(x) = 8x + 2 \) and \( h'(x) = 2 \). So

\[
\frac{d}{dx} \left( \frac{g(x)}{h(x)} \right) = \frac{h(x) \frac{d}{dx} g(x) - g(x) \frac{d}{dx} h(x)}{[h(x)]^2} = \frac{(5 + 2x) \left( \frac{d}{dx}(4x^2 + 2x - 3) - (4x^2 + 2x - 3) \frac{d}{dx}(5 + 2x) \right)}{(5 + 2x)^2} - \frac{(5 + 2x)(8x + 2) - (4x^2 + 2x - 3)(2)}{(5 + 2x)^2}
\]

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