\[ \frac{1}{4} (2)^4 - \frac{5(2)^2}{2} + C, \quad 4 = 4 - 10 + C, \quad C = 10. \]
\[ f(x) = \frac{1}{4}x^4 - \frac{5x^2}{2} + 10. \]

### 8.2 Segment: Integration by Substitution

The chain rule for derivatives states that the derivative of \( f(u(x)) \) with respect to \( x \) is given by \( f'(u) \cdot u'(x) \); \( \frac{df(u(x))}{dx} = f'(u) \cdot u'(x) \). We can turn this into the substitution rule for integrals: \( f(u(x)) = \int \frac{df(u(x))}{dx} \, dx = \int f'(u) \cdot u'(x) \, dx. \)

**Example 5**

\[ \int \sqrt{4x^2 - x} \ (8x - 1) \, dx \]
Let \( u(x) = 4x^2 - x \) then \( u'(x) = 8x - 1 \) and \( \int \sqrt{4x^2 - x} \ (8x - 1) \, dx = \int \sqrt{u} \cdot u'(x) \, dx \). Now \( \int \sqrt{u} \, du = \int u^{1/2} \, du = \frac{u^{3/2}}{3/2} + C = \frac{2u^{3/2}}{3} + C \), so
\[
\int \sqrt{4x^2 - x} \ (8x - 1) \, dx = \frac{2(4x^2-x)^{3/2}}{3} + C.
\]

**Example 6**

\[ \int x^2e^{x^3} \, dx \]
Let \( u(x) = x^3 \) then \( u'(x) = 3x^2 \) and \( \int x^2e^{x^3} \, dx = \frac{1}{3} \int 3x^2e^{x^3} \, dx = \frac{1}{3} \int e^u u'(x) \, dx = \frac{1}{3} e^u + C = \frac{1}{3} e^{x^3} + C. \)