Solutions: Solving Inequalities; Increasing, Decreasing Functions

**1.** For the function \( y = 2x^2 + 11x + 5 \), solve the inequality \( y \leq 0 \).
   
   First: \( 2x^2 + 11x + 5 = 0, \ (2x + 1)(x + 5) = 0, \ x = -\frac{1}{2}, 5 \)
   
   Second: Pick \( x \) values between the values found in the first step.
   
   Choose \(-10, -1, 0\) and \(0\)

   Third: Test the \( x \) values in the original inequality \( 2x^2 + 11x + 5 \leq 0 \).
   
   \( 2(-10)^2 + 11(-10) + 5 = 95 > 0 \)
   
   \( 2(-1)^2 + 11(-1) + 5 = -4 < 0 \)
   
   \( 2(0)^2 + 11(0) + 5 = 5 > 0 \)

   The set of intervals that contain the solutions to \( 2x^2 + 11x + 5 \leq 0 \) is just the interval containing \(-1\) and the endpoints; the interval from \(-5\) up to \(-1/2\): \([-5, -1/2]\)

**2.** For the function \( y = \frac{x-3}{5x+1} \), solve the inequality \( y \leq 0 \).
   
   First: \( \frac{x-3}{5x+1} = 0, \ x - 3 = 0, \ x = 3 \); also, \( \frac{x-3}{5x} \) is undefined when \( 5 + x = 0, \ x = -5 \)
   
   Second: Pick \( x \) values between the values found in the first step.
   
   Choose \(-10, 0, 10\)

   Third: Test the \( x \) values in the original inequality \( \frac{x-3}{5x} \leq 0 \).
   
   \( \frac{x-3}{5-10} = \frac{-10-3}{-10} = \frac{13}{5} > 0 \)
   
   \( \frac{x-3}{5-3} = \frac{0-3}{2} = -\frac{3}{2} < 0 \)
   
   \( \frac{x-3}{5+10} = \frac{10-3}{15} = \frac{7}{15} > 0 \)

   The set of intervals that contain the solutions to \( \frac{x-3}{5x} \leq 0 \) is just the interval containing \(0\) and the endpoint \(3\); the interval from \(-5\) up to \(3\): \((-5, 3]\)

**3.** For the function \( y = 4x + 16 \), solve the inequality \( y \geq 0 \).
   
   First: \( 4x + 16 = 0, \ x = -4 \)
   
   Second: Pick \( x \) values between the values found in the first step.
   
   Choose \(-10, 0\)

   Third: Test the \( x \) values in the original inequality \( 4x + 16 \geq 0 \).
   
   \( 4(-10) + 16 = -24 < 0 \)
   
   \( 4(0) + 16 = 16 > 0 \)

   The set of intervals that contain the solutions to \( 4x + 16 \geq 0 \) is just the interval containing \(0\) and the endpoints; the interval from \(-4\) up to infinity: \([-4, \infty)\)

**4.** For \( y = x^2 - x - 12 \),

In order to determine when \( y \) is positive and when \( y \) is negative we need to proceed the in the same manner as we did for problems 1 through 5.

First: \( x^2 - x - 12 = 0, \ (x - 4)(x + 3) = 0, \ x = -3, 4 \)

Second: Pick \( x \) values between the values found in the first step.

Choose \(-10, 0, 10\)
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Third: Test the $x$ values in the expression $2x^2 + 11x + 5$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$2x^2 + 11x + 5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-10$</td>
<td>$98 &gt; 0$</td>
</tr>
<tr>
<td>$0$</td>
<td>$-12 &lt; 0$</td>
</tr>
<tr>
<td>$10$</td>
<td>$78 &gt; 0$</td>
</tr>
</tbody>
</table>

- a. find the values of $x$ for which $y$ is positive,
  - $y$ is positive for $x$ belonging to $(-\infty, -3) \cup (4, \infty)$
- b. find the values of $x$ for which $y$ is negative.
  - $y$ is negative for $x$ belonging to $(-3, 4)$

5. A function is given by the following graph. For which values of $x$ is the function increasing and for which values of $x$ is the function decreasing?

- Increasing: $(-\infty, -2), (1, \infty)$
- Decreasing: $(-2, 1)$

6. A function is given by the following graph. For which values of $x$ is the function increasing and for which values of $x$ is the function decreasing?

- Increasing: $(-\infty, 0), (2, \infty)$
- Decreasing: $(0, 1), (1, 2)$

7. A function is given by the following graph. For which values of $x$ is the function increasing and for which values of $x$ is the function decreasing?

- Increasing: $(-\infty, \infty)$
- Decreasing: Nowhere
Sketch the graphs of the following functions. Note where they are increasing and decreasing.

8. \( f(x) = \frac{1}{x-4} \)
   - Increasing: Nowhere
   - Decreasing: \((-\infty, 4), (4, \infty)\)

9. \( f(x) = \sqrt{x + 5} \)
   - Increasing: \([-5, \infty)\)
   - Decreasing: Nowhere

10. \( f(x) = \begin{cases} 
        x - 1, & x \leq 2 \\
        (x - 2)^3 + 1, & x > 2 
        \end{cases} \)
    - Increasing: \((-\infty, \infty)\)
    - Decreasing: Nowhere

11. \( f(x) = \begin{cases} 
        2 - x, & x < 0 \\
        2 - x^3, & x \geq 0 
        \end{cases} \)
    - Increasing: Nowhere
    - Decreasing: \((-\infty, \infty)\)

Using inequality tests on the derivatives, find where the functions below are increasing and decreasing.

12. \( f(x) = -2x^2 + 4x + 3 \)
    \( f'(x) = -4x + 4 \).
    Solve \( 0 = -4x + 4, x = 1 \). The solution value separates the real numbers into two intervals: \((-\infty, 1)\) and \((1, \infty)\). Test a value in each interval; choose \( x = 0 \) and
The solution values separate the possible number of tickets into two intervals: \((-\infty, 6)\) and \((6, \infty)\). Test a value in each interval; choose \(x = 1\) and \(x = 6.\) The function \(f'(x) = 2x + 8\) is increasing on the interval containing \(0: (-4, \infty).\) The function \(f(x) = x^2 + 8x + 10\) is decreasing on the interval containing \(-10: (-\infty, -4).\)

13. \(f(x) = x^2 + 8x + 10\)
\[ f'(x) = 2x + 8. \] Solve \(0 = 2x + 8, x = -4.\) The solution value separates the real numbers into two intervals: \((-\infty, -4)\) and \((-4, \infty).\) Test a value in each interval; choose \(x = -10\) and \(x = 0.\) \(f'(-10) = 2(-10) + 4 < 0, f'(0) = 2(0) + 4 > 0.\) The function \(f(x) = x^2 + 8x + 10\) is increasing on the interval containing \(0: (-4, \infty).\) The function \(f(x) = x^2 + 8x + 10\) is decreasing on the interval containing \(-10: (-\infty, -4).\)

14. \(f(x) = x^4 - 2x^3\)
\[ f'(x) = 4x^3 - 6x^2. \] Solve \(0 = 4x^3 - 6x^2, 0 = 2x^2(2x - 3), x = 0, \frac{3}{2}.\) The solution values separate the real numbers into three intervals: \((-\infty, 0), (0, 3/2)\) and \((3/2, \infty).\) Test a value in each interval; choose \(x = -1, x = 1\) and \(x = 2.\) \(f'(-1) = 4(-1)^3 - 6(-1)^2 < 0, f'(1) = 4(1)^3 - 6(1)^2 < 0, f'(2) = 4(2)^3 - 6(2)^2 > 0.\) The function \(f(x) = x^4 - 2x^3\) is increasing on the interval containing \(2: (3/2, \infty).\) The function \(f(x) = x^4 - 2x^3\) is decreasing on the intervals containing \(-1\) and \(1: (-\infty, 0), (0, 3/2).\)

15. \(f(x) = x^2\)
\[ f'(x) = \frac{2}{3}x^{1/3} = \frac{2}{3\sqrt[3]{x}}. \] Solve \(0 = \frac{2}{3\sqrt[3]{x}}, \) there are no solutions. The derivative is undefined when \(3x^{1/3} = 0, x = 0.\) The solution value separates the real numbers into two intervals: \((-\infty, 0)\) and \((0, \infty).\) Test a value in each interval; choose \(x = -1\) and \(x = 1.\) \(f'(-1) = \frac{2}{3\sqrt[3]{-1}} < 0, f'(1) = \frac{2}{3\sqrt[3]{1}} > 0.\) The function \(f(x) = x^{2/3}\) is increasing on the interval containing \(1: (0, \infty).\) The function \(f(x) = x^{2/3}\) is decreasing on the interval containing \(-1: (-\infty, 0).\)

16. \(f(x) = -5x - 7\)
\[ f'(x) = -5. \] Solve \(0 = -5, \) there are no solutions. The sign of \(f(x)\) will remain constant on the entire set of real numbers. Test a value; choose \(x = 0. f'(0) = -5 < 0.\) The function \(f(x) = -5x - 7\) never increases. The function \(f(x) = -5x - 7\) is decreasing on the interval \((-\infty, \infty).\)

17. The position \(s\) of a bouncing ball at time \(t\) is given by the function \(s = 96t - 16t^2,\) for \(0 \leq t \leq 6.\) Find the time interval in which the ball is rising and the time interval in which it is falling. \(s'(t) = 96 - 32t. \) Solve \(0 = 96 - 32t, t = 3.\) The solution value separates the time interval \(0 \leq t \leq 6\) into two intervals: \([0, 3)\) and \((3, 6].\) Test a value in each interval; choose \(t = 1\) and \(t = 4.\) \(s'(1) = 96 - 32(1) > 0, s'(4) = 96 - 32(4) < 0.\) The ball is rising on the interval \([0, 3).\) The ball is falling on the interval \((3, 6].\)

18. The profit \(p\) made by an opera company selling \(x\) tickets can be modeled by \(p = -2.4x + \frac{x^2}{3000} - 450.\) Where is \(p\) increasing and where is it decreasing? \(p'(x) = -2.4 + \frac{x}{1500}. \) Solve \(0 = -2.4 + \frac{x}{1500}, x = 3600.\) Since \(x\) cannot be negative, the solution value separates the possible number of tickets into two intervals: \([0, 3600)\) and \((3600, \infty).\) Test a value in each interval; choose \(x = 1\) and \(x = 4500.\) \(p'(1) = -2.4 + \frac{1}{1500} < 0, p'(4500) = -2.4 + \frac{4500}{1500} > 0.\) The profit \(p = -2.4x + \frac{x^2}{3000} - 450\) is increasing on the interval containing \(4500: (3600, \infty).\) The profit \(p = -2.4x + \frac{x^2}{3000} - 450\) is decreasing on the interval containing \(1: [0, 3600).\)