Curvature and Torsion Problem

We consider in this problem the space curve given by \( \mathbf{\beta}(t) = (2 \cos(2t), 3 \sin(t), 2t^2) \).

a. Use Maple to compute the curvature of this curve. It is easiest to use the procedures from the Maple document posted on the course site or the procedures presented in the text.

Note: \textit{Mathematica} calculates the curvature of this curve to be:

\[
\text{Curvature}[f_\_] := \\
\quad \text{Module}[\{df, d2f, ds\}, \\
\quad \quad df = D[f, t]; \\
\quad \quad d2f = D[df, t]; \\
\quad \quad ds = \text{Sqrt}[df \cdot df]; \\
\quad \quad \kappa = \text{FullSimplify}[\text{Sqrt}[\text{Cross}[df, d2f] \cdot \text{Cross}[df, d2f]] / ds^3]; \\
\quad \quad \kappa[t_] := \text{Evaluate}[\kappa]; \\
\quad \quad \text{Clear}[\kappa]] \\
\quad f[t_] := (2 \cos[2t], 3 \sin[t], 2t^2) \\
\quad \text{Curvature}[f[t]]; \kappa[t] \\
\frac{4 \sqrt{36 \cos[t]^6 + 9 (\cos[t] + t \sin[t])^2 + 16 (-2t \cos[2t] + \sin[2t])^2}}{(9 \cos[t]^2 + 16 (t^2 + \sin[2t]^2))^{3/2}}
\]

b. Use Maple to compute the torsion of this curve.

Note: \textit{Mathematica} computes the torsion to be:

\[
\text{Torsion}[f_\_] := \text{Module}[\{df, d2f, d3f, ds\}, \\
\quad df = D[f, t]; \\
\quad d2f = D[df, t]; \\
\quad d3f = D[d2f, t]; \\
\quad ds = \text{Sqrt}[df \cdot df]; \\
\quad \tau = \text{FullSimplify}[\text{Cross}[df, d2f] \cdot d3f / (\text{Cross}[df, d2f] \cdot \text{Cross}[df, d2f])]; \\
\quad \tau[t_] := \text{Evaluate}[\tau]; \\
\quad \text{Clear}[\tau]] \\
\quad Torsion[f[t]]; \tau[t] \\
\frac{-(24 \cos[t] (2t (-2 + \cos[2t]) - 3 \sin[2t]))/((190 + 292 t^2 + 9 (19 - 4 t^2) \cos[2t] + 2 (-5 + 128 t^2) \cos[4t] + 9 \cos[6t] + 72 t \sin[2t] - 256 t \sin[4t]))}{(190 + 292 t^2 + 9 (19 - 4 t^2) \cos[2t] + 2 (-5 + 128 t^2) \cos[4t] + 9 \cos[6t] + 72 t \sin[2t] - 256 t \sin[4t])^{3/2}}
\]

c. Plot the curvature and torsion functions over the interval \([0, 5]\). \textit{Briefly} comment on what you may expect to see the curve \( \mathbf{\beta} \) to do over this interval. Don't plot the functions on the same coordinate system; i.e. generate two plots. Does the torsion become undefined anywhere? What appears to happen to the curvature for large values of the parameter? What about the torsion for large values of the parameter?

d. Generate a plot of \( \mathbf{\beta}(t) \) over the interval \([0, 5]\). Make sure that the curve is clearly shown. (\textit{Mathematica} gives the plot below, after a little work.)
Plane Evolute Problem

Consider the curve \( \alpha(t) = \left( 2\cos(t) \sqrt{\cos(2t)} , 2\sin(t) \sqrt{\cos(2t)} , 0 \right) \). Plot this curve along with its plane evolute. Also plot the involute of this curve.